Writing Sample: Multi-Agent Training for Open Domains without Verifiers
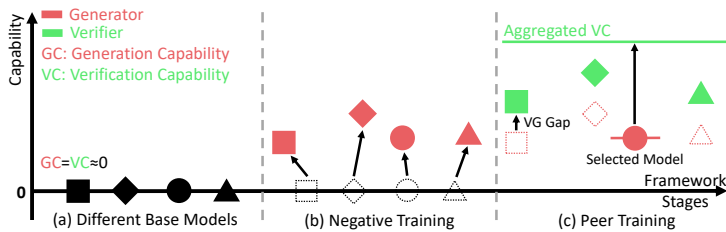## Cooperative AI PhD Fellowship Research Proposal
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Overview. Current research in reinforcement learning (RL) for language models has shown substantial progress in verifiable domains where oracle rewards are available (e.g., AIME). However, the recent success of strong frontier models such as Gemini [1] on IMO 2025 is shifting attention towards open-ended tasks where the verifiability of answers is not guaranteed [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Open domains at this level of difficulty have a fundamental difference: there exists no strong capable "teacher" model to provide reward signals. In this research proposal, I address the problem through a multi-agent training framework where models of comparable performance collaboratively improve each other, while also examining the "no free lunch (NFL)" problem and misalignment.

Problem Statement. Open-ended tasks have numerous solutions, making oracle verification difficult during RL even with an LLM judge or reference solution. These tasks span a wide range from generating math proofs [2, 3, 4, 5, 6, 7, 8, 9, 10, 11] to outputting harmless answers against jailbreak attacks [12, 13]. Most training methods *that scale* in open domains rely on a better capable "teacher" model to provide reference answers or reward signals. For example, methods that compare generations to reference solutions [2, 3, 4] require a model capable of creating the solutions beforehand. Similarly, methods that evaluate generations against handmade rubrics rely on a capable LLM judge for evaluation [5]. Also, using trained reward models implies access to a capable verifier. In contrast, some tasks are so difficult that there exists no capable verifier to provide training signals (e.g., proof for the Riemann Hypothesis). Current methods that try to train without a verifier aggregate online rollouts to approximate a reference solution or use model confidence as a reward [6, 7, 8, 9, 10, 11]. While such work on inventing new methodologies is important, no work tries to investigate open-ended generation from a more scientifically fundamental perspective on model capability: On extremely hard tasks where no model has the capability to generate or verify, how can we improve models? Answering this question is critical because benchmarks (e.g., IMO-Bench [14]) are rapidly approaching the difficulty of real-world tasks where models struggle. Traditional RL approaches that rely on rewards from better capable models may become infeasible. Through this proposal, I outline two interrelated research questions for harnessing the power of available low-capable models: (1) *How do we give models generation capabilities?* and (2) *How do we give models verification capabilities?* In the process of addressing these questions, I build a multi-agent training framework that trains each model to be both a generator and verifier to improve itself and help the improvement of other peer models. Furthermore, I address the no free lunch problem which is likely to occur when training without external signals.

The Multi-Agent Framework
Motivation of Framework. Reinforcement learning requires both generation and verification capabilities: without a verifier, there will be no reward, and even with a verifier, there will be no reliable training signal if the generation capability is so poor that rewards are near zero. With only access to low capable models that lack both generation and verification capabilities, how do we train? More specifically, (1) how do we induce minimal generation capabilities to kick-start training and (2) how do we induce verification capabilities to continue training? This framework is built in two stages by answering each question.

Figure 1: **The Multi-Agent Training Framework.** Different base models initially lack generation and verification capabilities on hard tasks. First, Negative Training pushes each model away from itself, giving minimal but diverse capabilities to kick-start training. Then, Peer Training uses other models to approximate a verifier by leveraging the verification–generation gap and model aggregation. The aggregated verifier exceeds the verification capability of individual models.

Problem Setup. For tasks where no existing model (e.g., Llama, Qwen, GPT, Claude) can reliably generate meaningful solutions or verify them, we effectively only have access to models of low capability. For research, this definition of "low capability" can be enforced by selecting a set of prompts where all models consistently fail over multiple rollouts. On this selected task, all models start with equally low capability (Figure 1.a).

RQ1: How do we induce minimal generation capabilities? The key assumption is that base models have poor generation capabilities. A natural way to improve models by leveraging this is to train away from the base policy. To do this, I would like to explore "negative training" where we treat base model rollouts as low reward generations (Figure 1.b). My work [12] demonstrates a form of verifier-based "negative training" where the model trains away from compliant behavior. To extend this to non-verifiable domains, I plan to implement a REINFORCE-style [15] objective with a reward of -1 to penalize all samples, providing gradients that steer the model away from itself. While such negative training will not provide gradients in the oracle direction or guarantee convergence, it can push the generation quality above trivial answers, enabling a verifier to produce meaningful reward signals.

RQ2: How do we induce verification capabilities? Although we lack access to a high-capable verifier, we do have access to several low-capable models which we can ensemble to approximate a verifier. There are three insights into why aggregating models would work even when individual models have no verification capabilities. One, we can apply "negative training" to every model, which by improving generation capabilities also strengthens the internal representations needed for verification. Furthermore, different base models (e.g., Qwen, Llama) possess different cognitive behaviors [16] which would lead to diverse capabilities across models. Two, aggregation methods such as majority voting [17, 18, 19] or model ensembles [20, 21] surpass the capability of individual models, which would synergize even more once negative training improves each model. Three, the verification-generation (VG) gap [22, 23] may help models of similar capability verify each other's generations. Thus, with multiple models available to approximate a verifier and negative training which can increase initial generation capabilities, I would like to integrate the two solutions and build a two stage framework for multi-agent training where each model generates rollouts for on-policy training and also provides rewards for other models. The two stages: (1) Models first undergo individual "negative training" (Figure 1.b) to receive initial updates in meaningful directions. (2) Next, models go through "peer training" (Figure 1.c), where at each step a selected model generates rollouts and receives aggregated rewards from other models (e.g., majority-voted LLM judge). Each step in peer training improves one model, which serves as a verifier at a latter step to help improve another model. This framework is a fundamental training guideline to help a system of models improve on hard open domains, rather than a recipe confined to a single task. During my PhD, I plan to study its applications to several open domains such as math proofs, scientific discovery, and creative writing, and understand the core design choices to make it effective.

Risk Factors (No Free Lunch and Misalignment). The two main risks when training without external signals is that there is no free lunch (NFL) and misalignment can occur. In such self-improvement frameworks [24, 25, 26, 27], it is unclear how a system of models can obtain new information to improve capabilities, and negative training can induce updates in any direction, becoming misaligned or degenerated. My work on safety (TARS [12]) provides a starting point to address these problems. I encountered a relevant issue where large open answer spaces in safety led to degeneration by losing reasoning capabilities. The solution was to add an auxiliary task-completion reward on a separate task to encourage reasoning, which helped improve reasoning capabilities on the main task. This shows that mixing in a verifiable signal on a different task can improve the capabilities of the non-verifiable task. I plan to extend this direction to the multi-agent framework by mixing in verifiable tasks during training. For example, if the main open domain task is to generate difficult math proofs, I will interleave math problems that are verifiable. The intuition is that the skills or abilities (e.g., sub-proofs, theorems) required to generate a difficult math proof overlap with the skills required to solve verifiable math problems. The verifiable problems will help utilize and connect those skills [23], which will in turn reinforce reasoning on the non-verifiable task. In this way, new information (e.g., skills) enters the system of models through the auxiliary verifiable task, addressing the NFL problem, while the verifiable reward also guides the model towards aligned and non-degenerate behavior.

Societal Impact. With difficult benchmarks such as IMO-Bench [14] sparking interest, future benchmarks may exceed the capability of existing models. The proposed framework provides a guideline to train a system of low capable models which initially cannot generate or verify solutions. This has significant societal implications. *One*, the framework would greatly expand the application of models to open domains (e.g., scientific discovery, law, medicine) that have no available verifier. Language models have been restricted to tasks that require some form of supervision, whether through curated data or external reward signals; this paradigm will change. *Two*, utilizing several small models to tackle open-ended domains has been underexplored. Recent work on math proofs leverage the strongest available models [28], but model capacity cannot scale infinitely. *Three*, the framework elevates AI safety to be more important. Since models will improve on their own, safeguards to prevent misalignment will be crucial. Overall, this framework of multi-agent training for open-ended tasks offers a promising and impactful agenda for better understanding the cooperation of low capable models.

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